Rob Blake

- Dynamical systems
- HW #1

Problem 2

First, notice that we can recover the first digit of any number by looking at where $\log_{10} x/\mathbb{Z}$ falls on the unit circle. A certain contiguous interval will contain all numbers beginning with 1, another interval will contain all numbers beginning with 2, and so on.

Second, notice that when we take the logarithm of our sequence, we get a dynamical system that describes a rotation of the circle: $\log_{10} 2^n = n \log_{10} 2$

Third, since $\log_{10} 2$ is irrational, this particular orbit is dense in the circle. This means that each contiguous region describing a certain leading digit must contain an infinite number of points. This follows from the fact that each leading digit region is a half closed interval in $\mathbb{R}$, and a finite number of points can not be dense in any interval of $\mathbb{R}$.

Problem 3

Closed: This proof is found in Katok and Hasselblatt. The infinite intersection of closed sets is a closed set by definition of a topological space. Therefore, consider the following definition of a ω-limit set: $\bigcap_{t_n=0}^{\infty} \overline{\bigcup_{x\in X, T\geq t_n}^{\infty} \varphi^T(x)}$. This set is closed by definition.

Invariant: This doesn't follow in general. Define $\varphi(x)$ as follows:

$$\varphi(x) = \begin{cases} \frac{1}{2}x & if \quad x \neq 0 \\ .9 & if \quad x = 0 \end{cases}$$

The limit point for this map is $\{0\}$, but $\varphi(0) \neq 0$.

If $\varphi(x)$ is smooth, we can use the flow box theorem to prove that the derivative at all ω-limit points is zero, which in turn gives us the result that our map is constant at these points.